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The Present Value Model of Stock Prices:  
Regression Tests and Monte Carlo Results

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The Present Value Model of Stock Prices:  
Regression Tests and Monte Carlo Results

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## ABSTRACT

The variance bounds tests of the present value model of stock prices are re-examined in this paper. A direct test of the model based on ordinary least squares estimation of a simple regression equation is proposed as an alternative and it is shown that this regression approach has several advantages over the variance bounds tests. This test is easily adapted to the important case in which the percentage changes in real dividends and real stock prices are stationary processes. The tests are applied to quarterly data for the Standard & Poor's Index of 500 Common Stocks and the results are much more conclusive than those obtained by previous tests.





# THE PRESENT VALUE MODEL OF STOCK PRICES: REGRESSION TESTS AND MONTE CARLO RESULTS

Shiller (1981a and 1981b) and LeRoy and Porter (1981) have tested the present value model of stock prices by examining the implicit restrictions of this model on the variation of stock prices. Their results suggest that actual stock prices vary too much to be consistent with this model. If we examine their results closely, we find that Shiller does not construct formal statistical tests of the model and that most of the tests in LeRoy and Porter are not statistically significant because the standard errors of the variance estimates are quite large. In addition, some critics have argued that the relevant time series are not covariance stationary, even after the removal of a time trend, and that the variance estimates are unreliable. This argument has been made by Kleidon (1982, 1984) and by Marsh and Merton (1983, 1984). An alternative would be to model the percentage changes in dividends, earnings, and stock prices as covariance stationary time series. Shiller (1981a) considers this alternative specification, but notes that it does not lead to tractable variance bounds for stock prices. In this paper, a direct regression test of the present value model is developed as an alternative to the variance bounds test, and the test is extended to handle the case in which the percentage changes in dividends, earnings, and stock prices are covariance stationary.

The present value model of stock prices has the following form:

$$P_t = E_t \left\{ \sum_{j=1}^{\infty} R^{-j} D_{t+j} \right\},$$

where  $P_t$  is the asset price at the end of period  $t$  and  $D_t$  is the dividend paid during period  $t$ .  $E_t$  is the conditional expectations operator, conditional on information available in period  $t$ .  $E$  without a subscript will represent the unconditional expectations operator.  $P_t$  and  $D_t$  are expressed in real terms. This simple expectations model follows from the expected real rate of return being constant and is generally associated with risk neutrality in asset pricing models, but some empirical researchers argue that expected real rates of return are approximately constant even in the presence of risk aversion.<sup>1</sup> If we let

$$P_t^* = \sum_{j=1}^{\infty} r^{-j} D_{t+j},$$

we observe that the model implies

$$(1) \quad P_t^* = P_t + \eta_t,$$

where  $\eta_t$  is the forecast error. The assumption of rational expectations (or market efficiency) requires that  $P_t$  and  $\eta_t$  be uncorrelated, so that we get

$$\text{Var}(P_t^*) = \text{Var}(P_t) + \text{Var}(\eta_t),$$

and we conclude that  $\text{Var}(P_t) \leq \text{Var}(P_t^*)$ . Thus, we have an upper bound on the variation of stock prices.

Two problems are encountered when one attempts a test of this bound on the variation of stock prices. First, the time series must be covariance stationary; that is, the unconditional means, variances, and covariances of the series must be finite and must not depend on time. Shiller removes a long-term trend from his series on dividends and prices and applies the variance restrictions to the detrended series. LeRoy and Porter argue that there are no apparent trends in their

adjusted series for earnings and prices and that further adjustments are not necessary. The second problem deals with the estimation of the variance of  $P_t^*$  (or a detrended  $P_t^*$ ), a series which is not observable. Shiller calculates this series recursively by assuming a value at the end of the sample period and then he computes a sample variance for the constructed series, but he does not develop a statistical test of the variance restrictions. LeRoy and Porter formulate and estimate bivariate time series models to compute the variances. This method requires the researcher to formulate a time series model and then test the variance bounds conditional on the formulated model. Singleton (1980) has shown that the variance of a series like  $P_t^*$  can be estimated in the frequency domain if we have a prior estimate for the discount factor. The hypothesis tests developed for these variance estimators will depend on the large sample distribution theory for variance estimators. It is well-known that these large sample distributions depend on the effects of fourth cumulants of the innovations of the process generating the observed time series, and for this reason the distributions typically used for hypothesis testing are not robust if we drop the assumption that the innovations are multivariate normal.<sup>2</sup> It is extremely difficult to estimate the standard errors of the variance estimates if we want to relax the assumption of normally distributed innovations (or the assumption that the fourth cumulants are all zero). The large sample distributions for many of the standard econometric estimators (such as ordinary least squares, generalized least squares, or instrumental variables) do not require the innovations or error terms to be normally distributed. For this



reason, the regression tests developed in the next section impose a less restrictive set of assumptions.

LeRoy and Porter developed several tests for the implied variance bounds. Even though their point estimates indicate rejection of the present value model, the confidence intervals are so large that the model is rejected in only a few of the cases examined. In a chapter of my dissertation (1982, Ch. 5), I calculated frequency domain variance estimators and found the point estimates for the variances of the price series to be many times greater than the estimates for the upper bounds, but the standard errors associated with these estimates are so large that the tests of the model are barely significant at the 5% level. It appears that these variances are not being estimated with much precision. More recently, Flavin (1983) has examined the finite sample properties of the variance estimators and has found that the resulting tests are biased against the null hypothesis. One can hardly argue that these tests of the present value model are conclusive. The alternative regression tests are developed in Section I, and the empirical results are presented in Section II. In Section III, I present the results of a Monte Carlo study which examines the finite sample properties of the proposed test statistics.

#### I. The Alternative Regression Test

In a note, Geweke (1980) has shown that regression tests of expectations models are more powerful than variance bounds tests. This testing procedure has been the one most frequently applied in tests of other expectations models (e.g., tests of forward rates as unbiased

predictors of spot rates in foreign exchange markets). If the series  $P_t^*$  were observable, then the relationship in equation (1), implied by the present value model, could be easily tested by a least squares regression of  $P_t^*$  on a constant and  $P_t$ . To calculate  $P_t^*$ , we need a prior estimate of the discount factor  $\beta$  and the entire realization of the dividend series ( $D_t$ ,  $t = 1, 2, \dots, \infty$ ). Consider the following estimate of  $P_t^*$  from a sample  $t = 1, \dots, T, T+1$ :

$$\begin{aligned}\hat{P}_T^* &= \beta D_{T+1} + \beta P_{T+1} \\ \hat{P}_{T-1}^* &= \beta \hat{P}_T^* + \beta D_T \\ &\vdots \\ \hat{P}_t^* &= \beta \hat{P}_{t+1}^* + \beta D_{t+1} \\ &\vdots \\ \hat{P}_1^* &= \beta \hat{P}_2^* + \beta D_2,\end{aligned}$$

where  $\beta$  is determined from a prior estimate. Under the null hypothesis of the present value model, we still have the result that  $E_t(\hat{P}_t^*) = P_t$  for  $t = 1, \dots, T$ . In fact, this construction of  $P_t^*$  is similar to Shiller's except that we use the last value of the stock price instead of a sample average. We again have a regression relation:

$$(2) \quad \hat{P}_t^* = a + bP_t + e_t,$$

where  $a$  and  $b$  should equal zero and one and  $e_t$  is a forecast error which should be uncorrelated with  $P_t$  according to the null hypothesis. The parameters  $a$  and  $b$  can be estimated consistently by ordinary least

squares (OLS), but the disadvantages of this approach are that we require a prior estimate of  $\rho$  and  $e_t$  is a serially correlated error term.<sup>3</sup> Hansen and Hodrick (1980) have shown that OLS produces reliable, but inefficient estimates and test statistics for expectations models of this form.

Another approach to testing the relationship in equation (1) is to restate it in terms of observable dividends and stock prices. First we rewrite the equation under the alternative hypothesis using the lag operator ( $L$ :  $Lx_t = x_{t-1}$ ):

$$\frac{\rho L^{-1}}{1-\rho L^{-1}} D_t = a + bP_t + \eta_t.$$

Now we multiply both sides of the equation by the filter  $\frac{1-\rho L^{-1}}{\rho L^{-1}}$  to get

$$(3) \quad D_t = a - bP_t + \frac{b}{\rho} P_{t-1} + u_t,$$

where  $u_t = (\frac{1}{\rho} L - 1)\eta_t$ . We now have an equation that is a function of observable time series and an error term, and we do not require a prior estimate of  $\rho$ , it becomes another parameter to estimate with  $a$  and  $b$ . The application of this filter is similar to the usual generalized least squares (GLS) correction for first-order autocorrelation in the error term, but this filter is unstable.<sup>4</sup> As a result,  $u_t$  is a serially uncorrelated error term. In the regression of  $P_t^*$  on  $P_t$ ,  $P_t$  is predetermined but not strictly exogenous, and it is well-known that the standard GLS correction for serial correlation does not produce consistent parameter estimates if the right-hand side variables are not strictly exogenous. In equation (3),  $P_t$  and  $u_t$  are correlated, and we must use an instrumental variables (IV) estimator.

Under the null hypothesis of the present value model,  $a = 0$ ,  $b = 1$ , and  $E_t(u_{t+j}) = 0$  for  $j \geq 0$ . Under the alternative hypothesis, we relax the restrictions on  $a$  and  $b$  and preserve the restriction on the error term. Hence,  $u_t$  should be uncorrelated with any variable dated  $t-1$  or earlier, and a natural set of instruments includes lagged dividends, lagged stock prices, and lagged values of variables which are important for predicting dividends. Lintner (1956) and Fama and Babiak (1968) have shown that earnings play an important role in the behavior of dividends over time; thus lagged earnings should be included as instrumental variables. The advantage of this second approach to the regression test is that we do not require a prior estimate of the discount factor and we do not need to estimate the  $P_t^*$  series.

If we want to estimate the parameters of equations (2) and (3), the relevant time series must have finite second moments, and this condition is normally satisfied by requiring that the time series be stationary, or at least covariance stationary.<sup>5</sup> This condition is necessary for estimating the variances in the variance bounds tests and it is also important for the OLS and IV estimators. One approach is to follow Shiller and remove a long-term trend from the data as follows:  $p_t = P_t(1+g)^{-t}$  and  $d_t = D_t(1+g)^{-t}$  where  $p_t$  and  $d_t$  are the respective detrended series. The discount factor for the model using detrended data becomes  $\gamma = \rho(1+g)$ , and the following equations can be derived for the detrended series:



$$(4) \quad \hat{p}_t^* = a + bp_t + e_t$$

$$(5) \quad d_t = a - bp_t + \frac{b}{\gamma} p_{t-1} + u_t,$$

where  $\hat{p}_t^*$  is computed by using  $\gamma$ , detrended dividends, and the terminal value of detrended stock prices. For equation (5), we must use an IV estimator and we still have the condition that  $E_{t-1}(u_t) = 0$ .

In the alternative specification, the dividend process is not mean-reverting. Here we assume that the percentage changes in dividends and stock prices as well as the price-dividend ratio  $(\frac{\Delta D}{D}, \frac{\Delta P}{P}, \frac{P}{D})$  are stationary. Shiller (1981a) notes that the terminal condition for the present value model is not necessarily satisfied if we assume that only  $\frac{\Delta D}{D}$  and  $\frac{\Delta P}{P}$  are stationary. If the terminal condition is not satisfied, we have the undesirable result that there is no solution for the stock price. Shiller imposes the terminal condition by requiring the price-dividend ratio to be stationary.<sup>6</sup> Kleidon, Marsh, and Merton have presented evidence that this kind of model for dividends is not rejected by the data in tests with Shiller's trend model as the alternative hypothesis. For the remainder of the paper, I focus on this specification of dividends and stock prices, but I include the tests on detrended data so that the results can be compared with Shiller's variance bounds tests. The relationship in equation (1) is now modified as follows:

$$(6) \quad \frac{P_t^*}{D_t} = a^* + b^* \frac{P_t}{D_t} + \frac{\eta_t}{D_t},$$

and again  $a^* = 0$  and  $b^* = 1$  for the present value model. To develop the regression test, we replace  $P_t^*$  in (6) with its estimate  $\hat{P}_t^*$ . To

derive the corresponding filtered equation, we apply the same filter as before to both sides of the equation after rewriting it as follows:

$$P_t^* = \frac{\beta L^{-1}}{1 - \beta L^{-1}} D_t = a^* D_t + b^* P_t + \eta_t$$

$$D_t = a^* \left( \frac{1}{\beta} D_{t-1} - D_t \right) + b^* \left( \frac{1}{\beta} P_{t-1} - P_t \right) + \left( \frac{1}{\beta} \eta_{t-1} - \eta_t \right)$$

$$D_t = a D_{t-1} - b P_t + \frac{b}{\beta} P_{t-1} + \frac{-\eta_t + \frac{1}{\beta} \eta_{t-1}}{1 + a^*},$$

where  $a = \frac{a^*}{\beta(1+a^*)}$  and  $b = \frac{b^*}{1+a^*}$ . Then the equation can be rewritten as a function of stationary time series plus an error term.

$$(7) \quad \frac{D_t}{D_{t-1}} = a - b \left( \frac{P_t}{D_{t-1}} \right) + \frac{b}{\beta} \left( \frac{P_{t-1}}{D_{t-1}} \right) + u_t,$$

where  $u_t = \frac{-\eta_t + \frac{1}{\beta} \eta_{t-1}}{(1+a^*)D_{t-1}}$ . Under the null hypothesis for the present value model,  $a = 0$  and  $b = 1$ . The error  $u_t$  again has the property that it is uncorrelated with variables dated  $t-1$  and earlier, and it is not serially correlated. Note that  $E(u_t (\frac{P_t}{D_{t-1}})) \neq 0$ , and again we must use an IV estimator.

In addition to separate tests on the coefficients  $a$  and  $b$ , we can use the OLS estimators and the IV estimators to construct joint tests of the restrictions for the present value model (joint tests that  $a = 0$  and  $b = 1$ ). These regression tests have several advantages over the variance bounds test. The variance bounds test is based on an inequality restriction which may contain a substantial amount of slack, whereas the regression test constitutes a direct test of the model. In the regression tests, we avoid the need to construct

hypothesis tests for variances which usually require additional restrictions on the distributional properties of the innovations. For the tests based on IV estimators, we eliminate the need to use a prior estimate for the discount factor. The regression tests described here are more in the spirit of the variance bounds tests than the previous regression tests of market efficiency in the finance literature. The regression tests of market efficiency have focused on regressions of rates of return on past information to see if there is any predictable variation in returns and the results indicate that there is very little. The regression tests derived in this section, like the variance bounds tests, are tests of the relation in equation (1), where the underlying issue is the use of stock prices as unbiased predictors of future dividends holding discount rates constant, not the predictability of the return series.

In constructing the tests based on OLS estimates of equations (4) and (6) and IV estimates of equations (5) and (7), we encounter several subtle econometric problems. The error terms in (4) and (6) will be serially correlated, at least under the null hypothesis, and the standard variance matrix for the OLS estimates will be inappropriate. To account for this problem, we apply Hansen's (1979) method for computing asymptotic variances:

$$\text{Var}\left(\begin{smallmatrix} a \\ b \end{smallmatrix}\right) = T(X'X)^{-1}S(X'X)^{-1}$$

where  $X$  is the  $(T \times 2)$  matrix containing a constant and the right hand side variable and  $S$  is the spectral density matrix of either  $(e_t, e_t p_t)$  or  $(e_t, e_t \frac{p_t}{D_t})$  evaluated at the zero frequency. The spectral

density matrices are estimated by using the Parzen procedure and a flat window as described in Nerlove, Grether and Carvalho (1978, pp. 67-68). Second order autoregressions are used to pre-whiten the series. This approach to estimating the variance matrix incorporates the fact that the stock price variable is not strictly exogenous in the equation and it allows for conditional heteroskedasticity in the error term.

Our theory tells us nothing about the variances of the forecast errors or the error terms of the equations; hence there is nothing in the theory that requires conditional variances to be constant. The OLS estimators for (4) and (6) are still consistent and the variance matrix for the estimators accounts for the possibility of conditional heteroskedasticity. If the conditional variance of the error term in either (5) or (7) is constant, then the appropriate estimator is the nonlinear two-stage least squares estimator of Amemiya (1974). Note that if the conditional variance of the error term is constant in either (5) or (7), then it cannot be constant in the other equation. To account for conditional heteroskedasticity in (5) and (7), we apply Hansen's (1982) generalized method of moments (GMM) estimator. Let  $\underline{u}$  be a vector of length  $T$  containing the residuals of either (5) or (7);  $T$  is the sample size. Let  $Z$  be the  $(T \times k)$  matrix containing the instrumental variables and  $\underline{z}_t'$  be the  $t$ 'th row of  $Z$ . For equation (5),  $\underline{z}_t' = (1, d_{t-1}, p_{t-1}, c_{t-1})$ , where  $c_{t-1}$  is the detrended earnings measure lagged one period. For equation (7), the instruments are the growth rates in dividends and earnings, the price-dividend ratio, and the price-earnings ratio, all lagged one period so that



$$\underline{z}'_t = (1, \frac{D_{t-1}}{D_{t-2}}, \frac{C_{t-1}}{C_{t-2}}, \frac{P_{t-1}}{D_{t-1}}, \frac{P_{t-1}}{C_{t-1}}),$$

where  $C_t$  is the earnings measure. Let  $\underline{\theta}' = (a, b, \gamma)$  for (5) or  $\underline{\theta}' = (a, b, \delta)$  for (7) and the GMM-IV estimator is computed by

$$\min_{\underline{\theta}} \ell = (\frac{1}{T} \underline{u}'Z)W_T^{-1}(\frac{1}{T} Z'\underline{u}),$$

where  $W_T$  is a weighting matrix which can be a function of the sample data. If the error term is conditionally heteroskedastic, then the optimal weighting matrix is  $E(u_{t-t}^2 z_t z_t')$ . In most cases, one must estimate  $W_T$ . The following two-step estimator which replaces  $W_T$  with a consistent estimate  $\hat{W}_T$  is asymptotically equivalent to the optimal estimator. First, estimate  $\underline{\theta}$  using a weighting matrix which produces initial consistent estimates; then use the consistent estimates  $\hat{\underline{\theta}}$  to form  $\hat{u}_t$  and estimate  $W_T$  from the sample moments:

$$\hat{W}_T = \frac{1}{T} \sum_{t=1}^T \hat{u}_{t-t}^2 z_t z_t'.$$

Finally, re-estimate  $\underline{\theta}$  using  $\hat{W}_T$ . For the initial weighting matrix, I use  $W_T = \frac{1}{T} Z'Z$  which produces the nonlinear two-stage least squares estimator. The covariance matrix for  $\underline{\theta}$  is computed as follows:

$$\text{Var}(\underline{\theta}) = T [ (\frac{\partial \underline{u}}{\partial \underline{\theta}})' Z \hat{W}_T^{-1} Z' (\frac{\partial \underline{u}}{\partial \underline{\theta}}) ]^{-1}.$$

Using both the OLS estimates and the GMM-IV estimates, we can test the restriction of the present value model ( $a = 0, b = 1$ ) by computing the following  $\chi^2$  statistic:

$$\chi^2_{(2)} = \begin{pmatrix} a \\ b-1 \end{pmatrix}' V^{-1} \begin{pmatrix} a \\ b-1 \end{pmatrix},$$

where  $V$  is the variance-covariance matrix for the estimates  $a$  and  $b$ . The test statistic has a large sample distribution that is Chi-squared with two degrees of freedom. An additional test for the model can be computed by using the criterion function for the GMM-IV estimators of (5) and (7). We restrict the parameters  $a$  and  $b$  to their respective values under the null hypothesis and estimate the discount factor ( $\beta$  or  $\gamma$ ). We can then test the model by applying Hansen's specification test in Lemma 4.2: the statistic  $T \min \ell$  should have a  $\chi^2$  distribution with degrees of freedom equal to the number of orthogonality conditions (the number of instrumental variables) minus the number of estimated parameters, in this case one. If the value of this test statistic is too large, then the model is rejected. This test is more in the spirit of the familiar regression tests of market efficiency to which I alluded earlier. If there is significant correlation between the error term  $u_t$  and the lagged variables used as instruments, then the test will indicate rejection of the present value model.

## II. Empirical Results

The present value model of stock prices is tested by using quarterly data for the Standard and Poor's Index of 500 Common Stocks for the period 1947 to the second quarter of 1983. In its publication Trade and Security Statistics, Standard and Poor's compiles quarterly data on its price index of 500 common stocks as well as indices on earnings and dividends for the companies included in the index.<sup>7</sup> The implicit price deflator for personal consumption expenditures is used to deflate the three series. To detrend the data for estimation of equations (4) and

(5), I have used a method similar to that used by Shiller. The following regressions are used to estimate the common growth trend for real dividends and real stock prices:

$$\ln D_t = a_1 + \ln(1+g_1) \cdot t,$$

$$\ln P_t = a_2 + \ln(1+g_2) \cdot t.$$

The two estimates,  $\ln(1+g_1)$  and  $\ln(1+g_2)$ , are averaged to form the OLS estimate of the common growth rate  $(1+g)$ , and this estimate is used to detrend both series. A separate trend regression is estimated for real earnings. For equations (6) and (7), it is not necessary to detrend the data because the effects of long-term trends or growth rates are removed when we form ratios.

The initial research effort focused on the GMM-IV estimation of the filtered regression equations (5) and (7). The results are quite extreme: the estimates for  $b$  range from  $-.033$  to  $.017$ , the  $t$  statistics for the tests that  $b = 1$  range from  $-411.20$  to  $-75.74$ , and the  $\chi^2_{(2)}$  statistics for the joint tests range from  $6173$  to  $212000$ . These extreme results indicate rejection of the present value model at the zero marginal significance level. In a subsequent Monte Carlo study, which is described in Section III, I found that there is an extreme finite sample bias in the GMM-IV estimates. For 100 simulations of a model in which stock prices satisfy the present value model, the GMM-IV estimates of  $b$  range from  $-.068$  to  $.1075$  and the  $\chi^2_{(2)}$  statistics for the joint test have extremely high values. The Monte Carlo results indicate that tests based on the GMM-IV estimation of  $a$  and  $b$  in the

filtered regression equations have an extreme bias against the present value model. The detailed results on the GMM-IV estimation of (5) and (7) are omitted for this reason. The Monte Carlo study does not reveal extreme biases for the other estimators and test statistics, and the results are reported in Tables I and II.

The results for the OLS estimation of equations (4) and (6) are presented in Table I. Both equations are estimated for a longer sample period 1947-83 and a more recent sample period 1960-83. The purpose of the second sample period is to show the results for a more recent period.<sup>8</sup> Because the results for the two sample periods are similar, the discussion will concentrate on the longer sample period. The prior estimates for the discount factor are computed from estimates of the average real return. Let  $\bar{r}_1$  be the sample mean of  $\frac{P_t + d_t}{P_{t-1}}$ , and the estimate of  $\gamma$  is the reciprocal of  $\bar{r}_1$ . Let  $\bar{r}_2$  be the sample mean of  $\frac{P_t + D_t}{P_{t-1}}$ , and the estimate of  $\beta$  is the reciprocal of  $\bar{r}_2$ . The estimated trend coefficients used for detrending the data in (4) are shown in the middle of the table. For the longer sample period, the trend coefficients for  $\ln D_t$  and  $\ln P_t$  are .004398 and .005628, respectively, so that the resulting estimate of  $(1+g)$  is 1.005026. The same trend coefficients are used for the data for the more recent sample period because the estimated trend coefficient over the shorter period is negative for real stock prices. This result alone indicates that the data fit the model poorly or that stock prices deviated substantially from their long term trend. The estimates for  $a$  and  $b$  in the longer sample period are 35.47 and -.02478, and the respective  $t$  statistics for  $a = 0$  and  $b = 1$  are



26.33 and -42.82. The separate  $t$  tests on  $a$  and  $b$  indicate rejection at extremely low marginal significance levels. The  $\chi^2_{(2)}$  statistic for the joint test is 5014 which also has an extremely low marginal significance level. In addition, the sample standard deviation of  $p_t$  is sixteen times greater than  $\hat{\sigma}(p_t^*)$ , the estimate of the theoretical upper bound. In Shiller's study,  $\hat{\sigma}(p_t)$  exceeds  $\hat{\sigma}(p_t^*)$  by a factor of five times for the S&P series and by a factor of thirteen for the Dow Jones Industrial Average. The results of the OLS tests on detrended data imply the same dramatic rejection of the present value model.

The results for equation (6) are contained in the bottom panel of Table I, and again we have dramatic rejection of the model. The OLS estimates of  $a$  and  $b$  are 73.34 and -.02092, and the respective  $t$  statistics for  $a = 0$  and  $b = 1$  are 6.53 and -10.02. The separate  $t$  tests are not quite as dramatic, but the results indicate rejection at extremely low significance levels. The  $\chi^2_{(2)}$  statistic for the joint test has an extreme value of 366, and the sample standard deviation of the price-dividend ratio exceeds the estimate of its upper bound by a factor of 1.67. Even though the violation of the upper bound is not dramatic, the tests based on the OLS estimates indicate overwhelming evidence against the model.

Despite the problems with the GMM-IV estimates of  $a$  and  $b$  in the filtered regression equation, we can restrict  $a$  and  $b$  to zero and one and compute a  $\chi^2$  test based on Hansen's specification error test. The results of these tests are contained in Table II. The top panel contains the results for equation (5) and the bottom panel contains the results for equation (7). The  $\chi^2$  tests are all significant at the

1% level except for the test associated with equation (7) for the shorter sample period, which is significant at the 2.5% level. Although these tests are significant at standard significance levels, the results are not as dramatic as those obtained with variance bounds tests or the OLS regression tests. As I pointed out earlier, this test is similar to the regression tests of market efficiency, and the results are similar. This test can also be criticized by observing that earnings in period  $t-1$  are being used to predict dividends and prices in period  $t$ , but earnings are normally reported with some lag.

### III. The Monte Carlo Simulations

The regression tests reported in Section II indicate dramatic rejection of the present value model. Kleidon and Flavin have presented evidence that the variance bounds tests can be biased against the present value model, and for this reason a Monte Carlo study of the estimators and test statistics developed in Section I has been performed. The purpose of the simulation study is to examine the performance of the test statistics when stock prices do in fact satisfy the present value model. I have already noted that the Monte Carlo study of the GMM-IV estimator indicates a severe bias against the model.

The simulated model for dividends and earnings has the following form:

$$\ln D_t - \ln D_{t-1} = a_1 - a_2 \ln D_{t-1} + a_2 \ln X_t + \varepsilon_{1t}$$

$$\ln X_t = a_3 + \ln X_{t-1} + \varepsilon_{2t},$$

where  $\epsilon_{1t}$  and  $\epsilon_{2t}$  are normally distributed and independent. The coefficient on earnings is restricted so that we get a steady state dividend payout ratio which does not depend on the level of earnings; in this model the steady-state dividend payout ratio is  $\exp(\frac{a_1}{a_2})$ . The form of the dividend equation is motivated by Lintner's well-known study of dividend policy. In the model, earnings follow a multiplicative random walk with drift, so that  $D_t/D_{t-1}$  and  $X_t/X_{t-1}$  are stationary time series. I then compute stock prices using the present value model:

$$\begin{aligned} P_t &= \sum_{j=1}^{\infty} R^j E_t(D_{t+j}) \\ &= \sum_{j=1}^{\infty} R^j D_t (1-a_2)^j X_t^{1-(1-a_2)^j} \exp\left\{\frac{a_1((1-a_2)^j-1)}{-a_2}\right\} \\ &\quad + a_2 a_3 \left( \sum_{k=1}^j (j+1-k)(1-a_2)^{k-1} \right) + \frac{1}{2} \sigma_1^2 \frac{(1-a_2)^{2j}-1}{(-a_2)(2-a_2)} \\ &\quad + \frac{1}{2} \sigma_2^2 \left[ (1-a_2)^2 \frac{(1-(1-a_2)^{2j})}{(1-(1-a_2)^2)} + 2(1-a_2) \frac{(1-(1-a_2)^j)}{(-a_2)} + j \right] \end{aligned}$$

where  $R$  is the discount factor and  $\sigma_1^2$  and  $\sigma_2^2$  are the respective variances of  $\epsilon_{1t}$  and  $\epsilon_{2t}$ . As  $j$  gets large, the terms in the summation are approaching:

$$\begin{aligned} R^j E_t(D_{t+j}) &= X_t \exp\left\{\left(\frac{a_1}{a_2}\right) + \frac{1}{2} \sigma_1^2 \left(\frac{1}{a_2(2-a_2)}\right)\right. \\ &\quad \left.+ \frac{1}{2} \sigma_2^2 \frac{(1-a_2)^2}{1-(1-a_2)^2} + \sigma_2^2 \left(\frac{1-a_2}{-a_2}\right)\right\} [R \exp\{a_3 + \frac{1}{2} \sigma_2^2\}]^j \end{aligned}$$

As an approximation for some large k, we have

$$\sum_{j=k+1}^{\infty} \beta^j E_t(D_{t+j}) = \frac{X_t^* (\beta \exp\{a_3 + \frac{1}{2}\sigma_2^2\})^{k+1}}{1 - \beta \exp\{a_3 + \frac{1}{2}\sigma_2^2\}},$$

$$\text{where } X_t^* = X_t \exp\left\{\left(\frac{a_1}{a_2}\right) + \frac{1}{2}\sigma_1^2 \left(\frac{1}{a_2(2-a_2)}\right) + \frac{1}{2}\sigma_2^2 \frac{(1-a_2)^2}{(1-(1-a_2)^2)} + \sigma_2^2 \left(\frac{1-a_2}{-a_2}\right)\right\}.$$

I experimented with values of 100, 200, and 300 for k and found that the stock price using k = 100 is extremely close to the value using k = 300.

To get simulated data that resemble actual dividends and earnings, I estimated the parameter values by fitting the model with seasonal dummies to actual data. (Earnings for the S&P 500 have not been negative during the post World War II period.) The following model was used in the simulations:

$$\ln D_t - \ln D_{t-1} = -0.3938 - .5811 \ln D_{t-1} + .5811 \ln X_t + \epsilon_{1t}$$

$$\ln X_t - \ln X_{t-1} = .0042 + \epsilon_{2t}$$

$$\text{Var}(\epsilon_{1t}) = .030032, \text{Var}(\epsilon_{2t}) = .006979,$$

where the coefficients for the dummy variables are omitted and each intercept represents the average of the quarterly intercepts. The model was simulated without the seasonality, and the  $\beta$  value was set at .9801625 to be consistent with average real returns on a quarterly basis.

The results of 1000 simulations are contained in Table III and there is some evidence of a finite sample bias in sample sizes of 145.

The mean of the 1000 estimates for the slope coefficient  $b$  is .9048, compared to the true mean of one. This suggests that there is a small downward bias in this estimator. But the slope coefficients range from .2563 to 2.596 and never have a value as low as those estimated from actual data. The large sample approximation for the  $t$  statistics is the standard normal distribution, and the results in the middle of Table III indicate that neither of the separate  $t$  tests has a bias against the present value model. In fact, the simulations indicate that the Type I error at various critical values is being overstated. The asymptotic  $\chi^2_{(2)}$  distribution, however, appears to be a poor approximation for the joint test on  $a$  and  $b$ . The critical value for the 5 percent significance level is 5.99, but 294 (or 29%) of the simulations produced test statistics exceeding 5.99. In fact, 189 (or 19%) of the simulations produced values that exceeded the critical value at 0.5 percent level. The maximum simulated value for the test statistic was 96.16. The large sample distribution is a poor approximation in this application, but the joint test statistic computed from actual data in the sample period of 145 observations is much greater than the maximum value produced by the simulations. The unusually large  $\chi^2_{(2)}$  statistics seem to be the result of large negative covariances between the estimates of  $a$  and  $b$ . In summary, the simulations indicate that despite a small finite sample bias in the estimates of  $a$  and  $b$ , the estimates and the separate  $t$  statistics are not biased against the present value model. The joint test based on the  $\chi^2_{(2)}$  statistic does have a bias against the model. One final observation concerns the sample standard



deviations of  $\frac{P_t^*}{D_t}$  and  $\frac{P_t}{D_t}$ . The sample standard deviation of  $\frac{P_t}{D_t}$  exceeds that of  $\frac{P_t^*}{D_t}$  in only 16 of the 1000 simulations, and it never exceeds the estimated upper bound by a factor of more than 1.16. In the sample of 145 actual observations for equation (6), the standard deviation of  $\frac{P_t}{D_t}$  exceeds that of  $\frac{P_t^*}{D_t}$  by a factor of 1.67.

In Table IV, I present the results of 1000 simulations of the  $\chi^2_{(4)}$  statistic with a and b restricted in the filtered equation. For this test, we first estimate  $\beta$  with the GMM estimator allowing for conditional heteroskedasticity in the error term. The test statistic under the null hypothesis is distributed asymptotically as a  $\chi^2$  with 4 degrees of freedom, and the asymptotic distribution appears to be a good approximation for the upper tail of the distribution.

#### IV. Conclusion

The results for the S&P 500 data suggest that the present value model of stock prices with rational expectations does not adequately describe the behavior of asset prices in the stock market. The alternative tests described here do not require some of the additional assumptions required to derive tests based on estimates of variances, and the results are much more conclusive than those previously obtained for tests of variance bounds. In addition, this test is modified to handle the case in which the percentage changes in real dividends and real stock prices are stationary and the results are found to be qualitatively the same.

There are several competing hypotheses in the literature which attempt to explain the results of the variance bounds tests.

Blanchard and Watson (1982) interpret Shiller's results as evidence of an arbitrary term in the rational expectations solution for stock prices, specifically a stochastic bubble. Another more appealing explanation is that risk aversion and varying real interest rates are responsible for the observed volatility of stock prices. Kleidon, Marsh, and Merton have argued that combining the present value model with nonstationary processes for dividends and stock prices is capable of explaining stock price variability, but the results presented here for equation (6) reject this explanation. These tests do not allow us to pinpoint the causes of stock price volatility, but we can conclude that some of the variation in stock prices is due to something other than the variation in dividends or the market's expectations of future dividends. Indeed, a casual reading of the financial press suggests that stock prices are quite sensitive to interest rate changes.

# FOOTNOTES

1. For a discussion of this model and other expectations models, see LeRoy's survey (1982).
2. For the asymptotic distribution of sample variances for time series, see Hannan (1970, pp. 209-12). In a footnote (footnote 15, p. 567), LeRoy and Porter recognize this subtle assumption in the derivation of their tests.
3. It is easy to show that the error term should be a first order autoregression that is stable in the forward direction.  

$$e_t = \rho e_{t+1} + \eta_{t+1}$$
 where  $\eta_{t+1}$  is a variable which is uncorrelated with variables known at time  $t$  or earlier including itself.
4. The root of  $(1-\rho L^{-1})$  lies inside the unit circle, but the filter is stable going in the forward direction.
5. This subtle point is frequently ignored in applied econometrics. To apply Hansen's GMM estimator for example, we need to have stationary time series.
6. In a comment on Shiller's paper, Copeland (1983) has argued that the long term growth rate for dividends may be changing. His argument is confined to the effect of a one-period change, but we must consider how the growth rate changes over time. When we assume that the percentage changes in dividend and stock prices are stationary, we normally think of a constant growth rate for the series. If the random growth rate for dividends is itself a stationary process, then the series  $(\frac{\Delta D}{D}, \frac{\Delta P}{P})$  will be stationary. If we assume that the random growth rate is not stationary, then we again run the risk of not satisfying the terminal condition for the present value model.
7. The data for dividends are reported as a twelve-month moving total. LeRoy and Porter obtained the data necessary to recompute the quarterly dividends and their numbers are available in a technical appendix to their original paper.
8. Also, the S&P index was expanded from 90 stocks to 500 stocks in 1957.

REFERENCES

- Takeshi Amemiya, "The Nonlinear Two-Stage Least-Square Estimator," Journal of Econometrics 2 (1974): 105-10.
- Olivier J. Blanchard and Mark W. Watson, "Bubbles, Rational Expectations, and Financial Markets," Paul Wachtel, Ed., Crisis in the Economic and Financial Structure (Lexington: D. C. Heath and Company, 1982).
- Basil L. Copeland, "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?: Comment," American Economic Review (March 1983): 234-35.
- Eugene F. Fama and Harvey Babiak, "Dividend Policy: An Empirical Analysis," Journal of the American Statistical Association (December 1968): 1132-61.
- Marjorie Flavin, "Excess Volatility in the Financial Markets: A Reassessment of the Empirical Evidence," Journal of Political Economy (December 1983): 929-56.
- John Geweke, "A Note on the Testable Implications of Expectations Models," University of Wisconsin-Madison, Revised November 1980, SSRI Workshop Series.
- Clive W. J. Granger, "Some Consequences of the Valuation Model When Expectations are Taken to be Optimum Forecasts," Journal of Finance (March 1975): 135-45.
- E. J. Hannan, Multiple Time Series (New York: John Wiley & Sons, 1970).
- Lars P. Hansen, "The Asymptotic Distribution of Least Squares Estimators with Endogenous Regressors and Dependent Residuals," Carnegie-Mellon University, March 1979.
- \_\_\_\_\_, "Large Sample Properties of Generalized Method of Moments Estimators," Econometrica (July 1982): 1029-54.
- \_\_\_\_\_ and Robert J. Hodrick, "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis," Journal of Political Economy (October 1980): 829-53.
- Allan W. Kleidon, "Bias in Small Sample Tests of Stock Price Rationality," Stanford University, May 1982.
- \_\_\_\_\_, "Variance Bounds Tests and Stock Price Valuation Models," Stanford University, June 1984.

Stephen F. LeRoy, "Expectations Models of Asset Prices: A Survey of Theory," Journal of Finance (March 1982): 185-217.

\_\_\_\_\_, "Efficiency and the Variability of Asset Prices," American Economic Review (May 1984): 183-87.

\_\_\_\_\_ and Richard D. Porter, "The Present-Value Relation: Tests Based on Implied Variance Bounds," Econometrica (May 1981): 555-74.

John Lintner, "Distribution of Incomes of Corporations Among Dividends, Retained Earnings, and Taxes," American Economic Review (May 1956): 97-113.

Robert Lucas, "Asset Prices in an Exchange Economy," Econometrica (November 1978): 1429-45.

Terry A. Marsh and Robert C. Merton, "Aggregate Dividend Behavior and Its Implications for Tests of Stock Market Rationality," Massachusetts Institute of Technology, September 1983.

\_\_\_\_\_, "Earnings Variability and Variance Bounds Tests for the Rationality of Stock Market Prices," Massachusetts Institute of Technology, March 1984.

Mark Nerlove, David M. Grether, and Jose L. Carvalho, Analysis of Economic Time Series: A Synthesis (New York: Academic Press, 1979).

Louis O. Scott, "Risk Aversion, Rational Expectations, and Constant Expected Real Rates of Return," Unpublished Ph.D. dissertation, Department of Economics, University of Virginia, 1982.

Robert J. Shiller, "The Use of Volatility Measures in Assessing Market Efficiency," Journal of Finance (May 1981a): 291-304.

\_\_\_\_\_, "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" American Economic Review (June 1981b): 421-36.

\_\_\_\_\_, "Reply," American Economic Review (March 1983): 236-37.

Kenneth J. Singleton, "Expectation Models of the Term Structure and Implied Variance Bounds," Journal of Political Economy (December 1980): 1159-76.



TABLE I

## OLS Models

	Sample Period <u>1947:I to 1983:II</u>	Sample Period <u>1960:I to 1983:II</u>
T	145	93
<u>Detrended Data:</u> $p_t^* = a + bp_t + e_t$		
$\gamma$ (Prior Estimate)	.9846	.9917
a	35.47 (1.347)	43.95 (3.611)
b	-.02478 (.02393)	.1790 (.05632)
$\sigma_e^2$	.8887	9.431
$\chi^2_{(2)}$	5014.13	305.91
<u>Trend Coefficients</u>		
Dividends	.004398	.004398
Stock Prices	.005628	.005628
Earnings	.005572	.005572
(1+g)	1.005026	1.005026
<u>Ratio Data:</u> $\frac{P_t^*}{D_t} = a + b \frac{P_t}{D_t} + e_t$		
R (Prior Estimate)	.9797	.9868
a	73.34 (11.24)	92.96 (9.979)
b	-.02092 (.1019)	-.04153 (.09948)
$\sigma_e^2$	277.1	69.52
$\chi^2_{(2)}$	366.27	128.84

Standard errors are in parentheses. The standard errors and  $\chi^2_{(2)}$  statistics are based on error terms with serial correlation and conditional heteroskedasticity.

TABLE II

## Tests of Restricted Filtered Equations

	Sample Period 1947:I to 1983:II	Sample Period 1960:I to 1983:II
<u>Detrended Data:</u> $d_t + p_t = \gamma p_{t-1} + u_t$		
T	145	93
$\hat{\gamma}$	.9876	.9881
$\hat{\sigma}_u^2$	14.46	30.16
D.W.	1.70	1.70
$T \min \ell = \chi^2_{(3)}(a=0, b=1)$	13.59	12.85

Trend coefficients are the same as in Table I.

<u>Ratio Data:</u> $\frac{D_t + P_t}{D_{t-1}} = \beta \frac{P_{t-1}}{D_{t-1}} + u_t$		
T	144	92
$\hat{\beta}$	.9823	.9904
$\hat{\sigma}_u^2$	60.05	73.61
D.W.	1.71	1.68
$T \min \ell = \chi^2_{(4)}(a=0, b=1)$	14.04	13.12

Estimates are based on conditional heteroskedasticity for error terms.

TABLE III

Results of 1000 Simulations of OLS Model  
Sample Sizes = 145

	<u>Mean</u>	<u>Range</u>
$\beta$ (Prior Estimate)	.9800	.9580 to 1.0002
a	2.787	-95.65 to 61.20
b	.9048	.2563 to 2.596
$t(a=0)$	.06366	-1.483 to 3.047
$t(b=1)$	.2699	-4.961 to 1.908
$\chi^2_{(2)}$	6.513	.000367 to 96.19

<u>x</u>	<u>Number of Times</u> <u><math> t(a)  &gt; x</math> (%)</u>		<u>Number of Times</u> <u><math> t(b=1)  &gt; x</math> (%)</u>		<u>PR[ <math> t  &gt; x</math> ]</u>
1.645	4	(.4%)	24	(2.4%)	.10
1.96	2	(.2%)	11	(1.1%)	.05
2.326	1	(.1%)	7	(.7%)	.025
2.576	1	(.1%)	4	(.4%)	.01

<u>x</u>	<u>Number of Times</u> <u><math>\chi^2_{(2)}</math> Exceeds x (%)</u>		<u>Pr[ <math>\chi^2_{(2)} &gt; x</math> ]</u>
5.99147	294	(29.4%)	.05
7.37776	247	(24.7%)	.025
9.21034	212	(21.2%)	.01
10.5966	189	(18.9%)	.005

TABLE IV

Results of 1000 Simulations for Restricted Filtered Model  
 Sample Sizes = 144

<u>Mean Values</u>		
$\bar{R}$	.9808	
Test Statistic, $\chi^2_{(4)}$	3.956	
<u>x</u>	Number of Times Test Statistic Exceeds x (%)	<u><math>\Pr[\chi^2_{(4)} &gt; x]</math></u>
9.4877	37 (3.7%)	.05
11.1433	19 (1.9%)	.025
13.2767	7 (.7%)	.01
14.8602	5 (.5%)	.005









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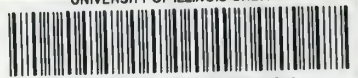


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